Heathfield Primary School

Calculation Policy

Our Mission Statement:

Learning toether, Learning for Life

Chair of Governors	Zulfi Jiva
Headteacher	Mark Thornley
Policy written by	Esha Seth (Maths Lead) April 2020
Date approved by governors	Summer 2020
Date of next review	April 2021
Committee Responsible	

INTRODUCTION

At Heathfield, our curriculum lays out the way we teach our children to achieve our mission statement; *Learning together, learning for life.*

The curriculum consists of all the planned activities and routines that we organise in order to promote learning, confidence and self-esteem. It includes not only the formal requirements of the National Curriculum, but also the range of extra-curricular activities that the school organises in order to enrich the experience of the children. The children at Heathfield are provided with a an inter connected curriculum that promotes meaningful connections between concepts and knowledge **(Heathfield Head),** develops genuine and robust character traits to prepare children for life in the modern world **(Heathfield Heart)** and opportunities for children to use and apply their Head and Heart to answer learning questions **(Heathfield Hands)** resulting in knowledgeable, physically and mentally healthy children that achieve their potential and have a solid foundation to become life-long learners.



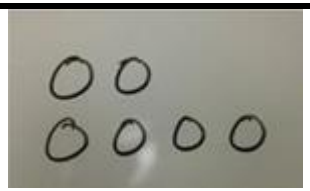
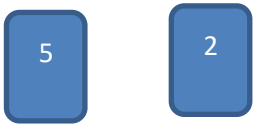
RATIONALE

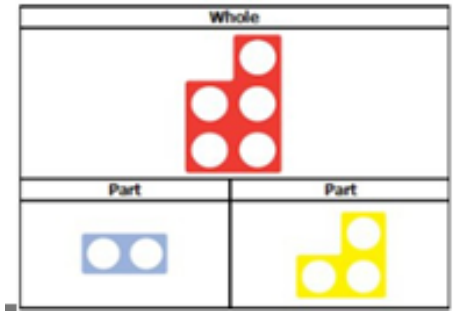
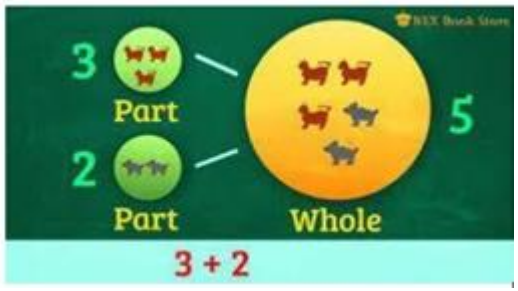
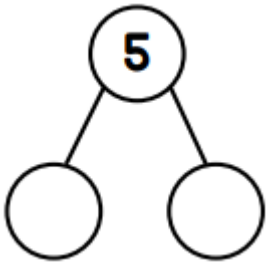
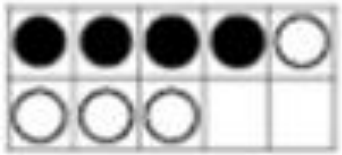
This policy has been largely adapted from the White Rose Calculation Policy with further material added. It is a working document and will be revised and amended as necessary. It aims to ensure consistent strategies, models and images are used across the school to embed and deepen children's learning and understanding of mathematical concepts. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. It has been written to ensure consistency and progression throughout the school.

The progression of strategies and written methods has been set out so that the children develop the understanding of the four operations. Our aim is to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task. Consistency in language is essential for pupils to understand the concepts presented in mathematics. For example, in terms of place values 'ONES' will be used consistently across the school. This will support children right from EYFS to KS2 to build on prior knowledge.

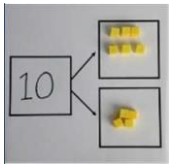
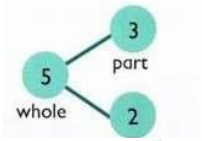
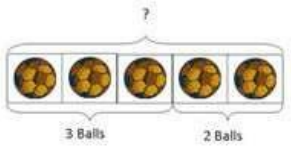
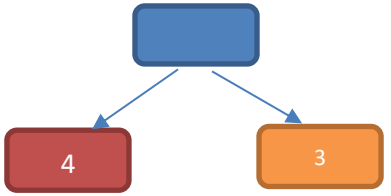
At Heathfield we follow the Concrete, Pictorial, Abstract (CPA) approach where children use concrete objects to help them make sense of the concept or problem; this could be anything from real or plastic fruit, to straws, counters or cubes. This is then developed by the use of images, models and children's own pictorial representations before moving on to the abstract mathematics. Children will travel along this continuum repeatedly, often revisiting previous stages when a concept is extended. It is also worth noting that if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. In fact, this is essential for the children to understanding the connection between them. Alternatively, if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial.
Similarly, although the strategies are taught in a progressive sequence, they are designed to equip children with a 'tool box' of skills and strategies that they can apply to solve problems in a range of contexts. Therefore, as a new strategy is taught it does not necessarily supersede the previous, but builds on prior learning to enable children

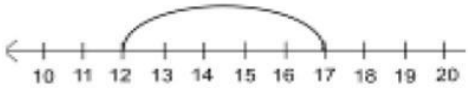
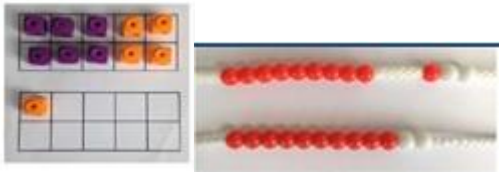
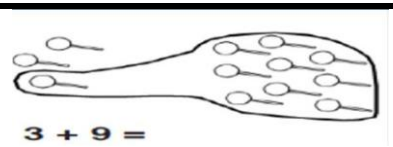
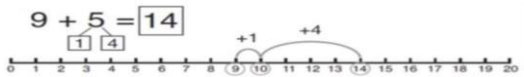
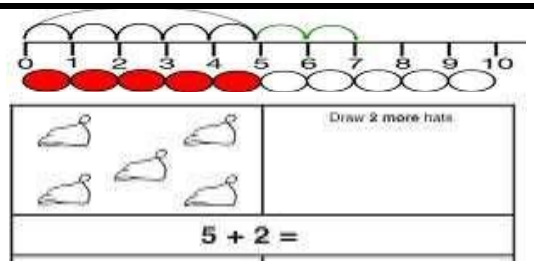
to have a variety of tools to select from. As children become increasingly independent, they will be able to and must be encouraged to select those strategies, which are most efficient for the task.
The strategies are separated into the four operations for ease of reference. However, it is intended that addition and subtraction, and multiplication and division will be taught together to ensure that children are making connections and seeing relationships in their mathematics. Our aim is to get each child to show fluency, reasoning and problem solving skills from EYFS – Year 6. Effective teaching of the strategies rely on increasing levels of number sense, fluency and ability to reason mathematically. Children must be supported to gain depth of understanding within the strategy through the CPA approach and not learn strategies as a procedure. The long-term aim is for children to be able to select an efficient method of their choice that is appropriate for a given task. They should do this by always asking themselves:
'Do I need to use manipulatives to help me?'
'Can I do this using drawings or jottings?'
'Do I need to use a written method?'
'Can I do this in my head?'

Reception - Addition			
Objective/strategy	**Concrete**	**Pictorial**	**Abstract**
Early learning goal: add and subtract two single-digit numbers and count on or back to find the answer			
Add two single digit numbers Count on	Use counters, cubes or other concrete resources to count out the correct amount for each number in the calculation and then combine them to find the total  		Use number cards to replace the images and this will help reinforce the use and recognition of numbers 

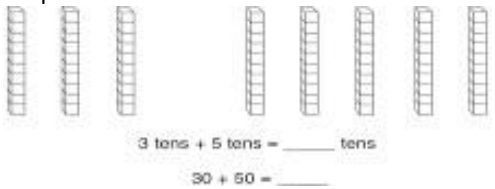
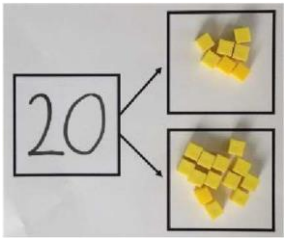
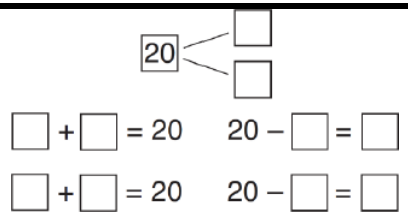
Part whole model			 2+3=5
Counting all	Tens frame with counters 	Pictures 	4+4=8

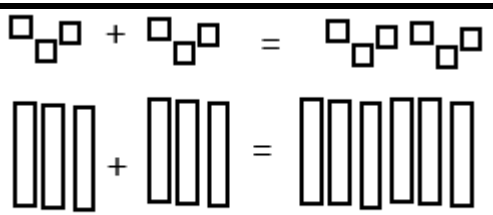
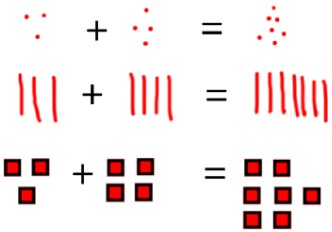
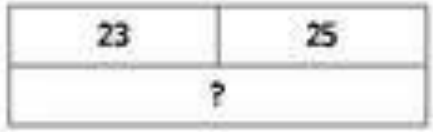
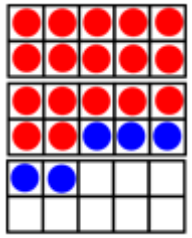
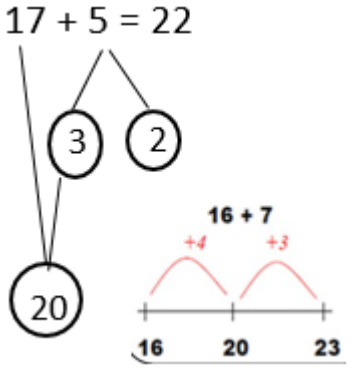
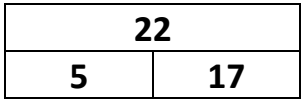
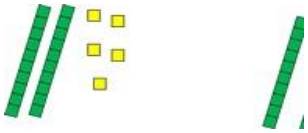
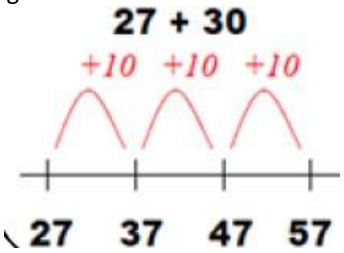
Year 1 - Addition

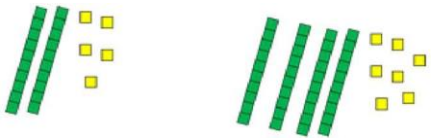
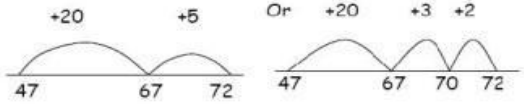
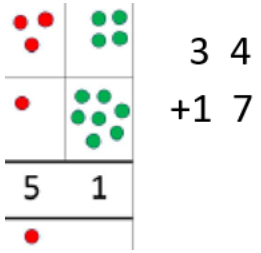
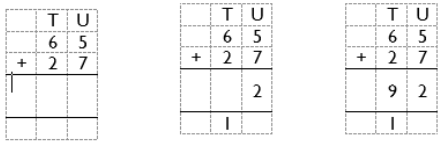
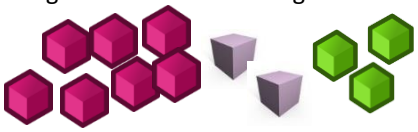
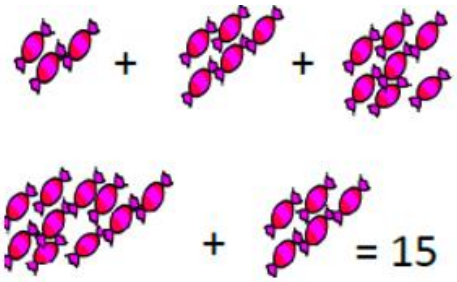
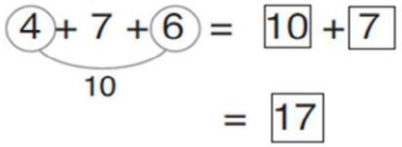
Objective/**Strategy**	**Concrete**	**Pictorial**	**Abstract**
End of Year Objective: Add one-digit and two-digit numbers to 20, including 0, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9			
Combining two parts to make a whole Part- whole model	Use cubes to add two numbers together as a group or in a bar.     Use part, part whole model.	Use pictures to add two numbers together as a group or in a bar.    	7 = 4 + 3 4 + 3 = 7  Include missing number questions to support varied fluency: 7 = ? + 3 4 + ? = 7
Starting at the bigger number and counting on Counting on	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer. 	12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

			
Regrouping to make 10.	6 + 5 = 11 Start with the bigger number and use the smaller number to make 10. Use ten frames. 	 Use pictures or a number line. Regroup or partition the smaller number using the part, part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do need to make 10? How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20 More than	 2 more than 5.		Include missing number questions: 8 = ? + 3 5 + ? = 8 Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

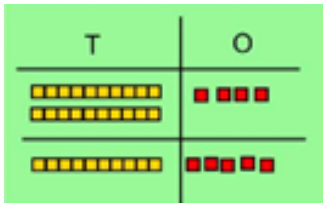
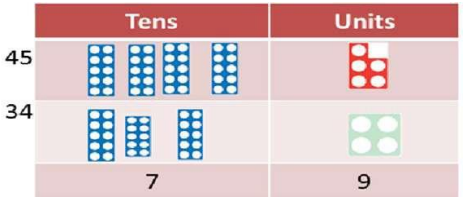
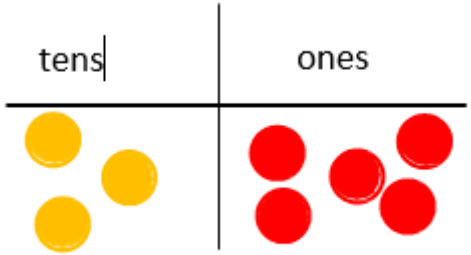
Year 2 - Addition

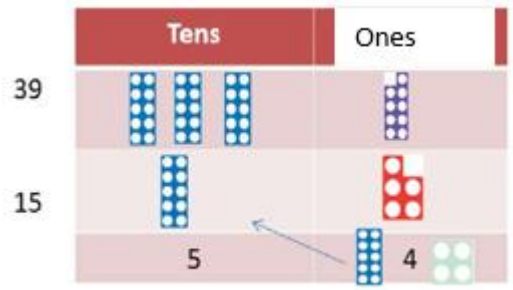
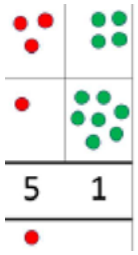
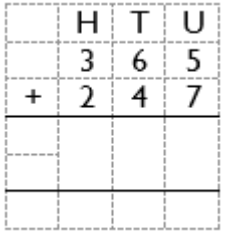
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers. Add numbers with up to three digits, using formal written method of columnar addition (For those who are ready to record in this way).			
Adding multiples of ten Dienes and beads	Model using dienes and bead strings 50= 30 = 20 	Use representations for base ten. 	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	Children explore ways of making numbers within 20 		□ + 1 = 16 16 – 1 = □ 1 + □ = 16 16 – □ = 1

Using known facts Partitioning/place value		Children draw representations of H,T and O 	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* 300 + 400 = 700
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48
Add a two digit number and ones	17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32 	Use part whole and number line method to model 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22 - 17 = 5 22 - 5 = 17 
Add a 2 digit number and tens	Children to explore and understand that when adding tens, ones does not change  25 + 10 = 35	Counting on in tens on a number line 	Using tens place value to find the missing number 27 + 10 = 37 27 + 20 = 47 27 + □ = 57

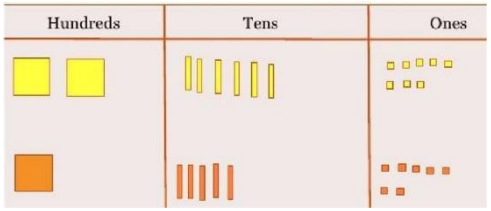
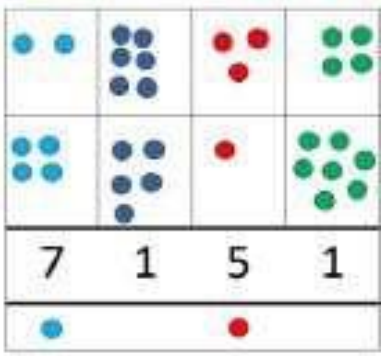
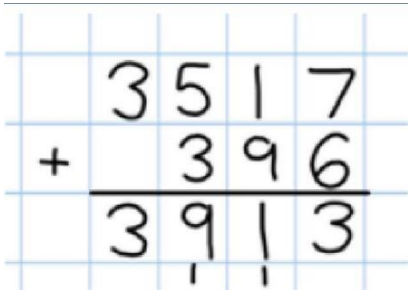
Add two 2-digit numbers	Model using dienes, place value counters and numicon 	Use number line and bridge ten using part whole if necessary.  Children can also draw a representation of the grid to further support their understanding, carrying the ten under the line 	Use partitioning initially then building on to column addition 
Add three 1-digit numbers	Combine to make 10 first if possible, or bridge 10 then add third digit 	Regroup and draw representation. 	Combine the two numbers that make/ bridge ten then add on the third. 

Year 3 - Addition

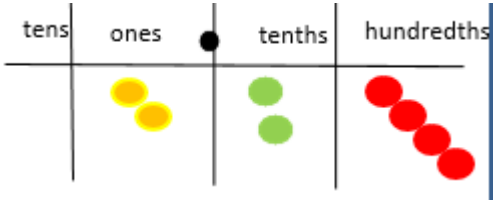
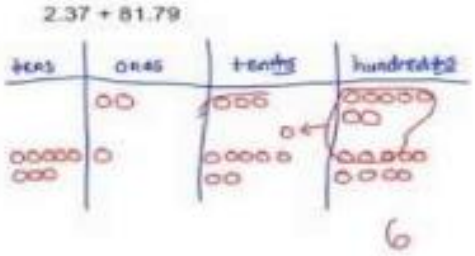
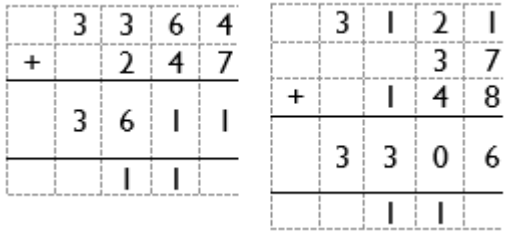
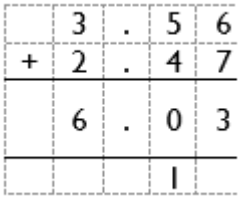
Objective/strategy	Concrete	Pictorial	Abstract
End of Year objective: Add numbers with up to three digits, using formal written method of columnar addition			
Column Addition—no regrouping	Dienes or numicon Add together the ones first, then the tens.   	Children move to drawing the counters using a tens and one frame.  	Add the ones first, then the tens, then the hundreds. 2 2 3 + 1 1 4 3 3 7
Column Addition with regrouping.	Exchange ten ones for a ten. Model using numicon and place value counters.	Children can draw a representation of the grid to further support their understanding, carrying the ten under the line	Use formal column method and children to understand the concept of regrouping as number is regrouped in the next place value. Use partitioning where required.

		 3 4 +1 7 5 1	

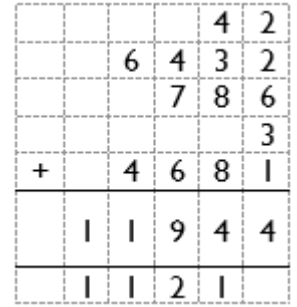
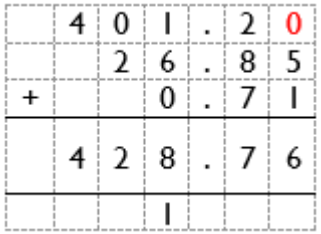
Year 4 - Addition

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Add numbers with up to 4 digits and decimals with one decimal place using the formal written method of columnar addition where appropriate.			
add numbers with up to 4 digits By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding: -several numbers with different numbers of digits, understanding the place value; -decimals with one decimal place, knowing that the decimal points line up under one another.	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	Draw representations using place value grid. 	Continue from previous work to carry hundreds as well as tens. Relate to money and measures. 

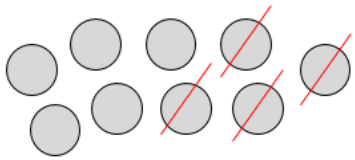
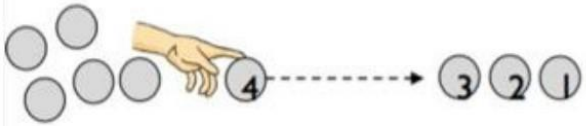
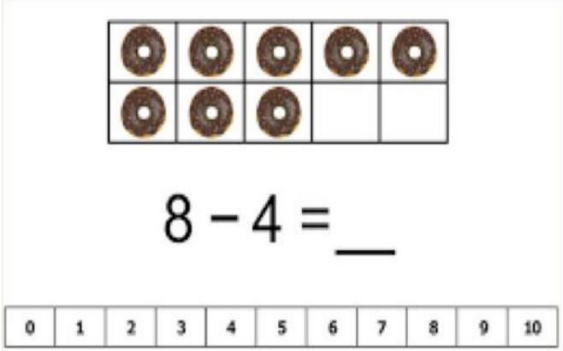
Year 5 - Addition

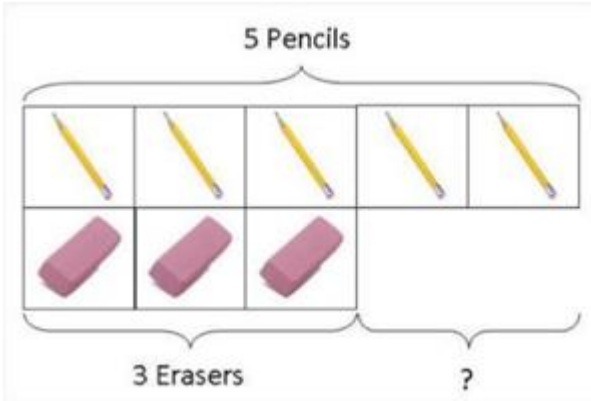
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Add whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar addition).			
Add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	Use concrete resources when necessary to build on from Year 4 Introduce decimal place value counters and model exchange for addition. 	Children draw their own representation to understand the place value of each digit when using decimals and adding starting from hundredths 	Children continue to use the carrying method to solve calculations  

Year 6 - Addition

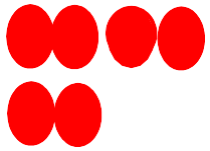
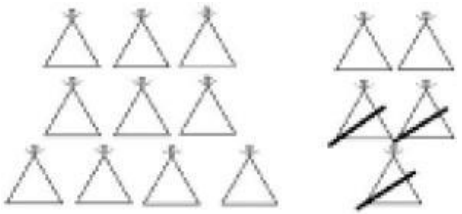
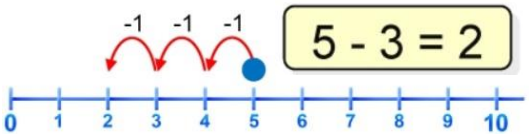
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Add whole numbers and decimals using formal written methods (columnar addition).			
add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.	Use variety of concrete resources as in Year 4 and 5 to embed understanding of concepts when adding numbers of increasing complexity	Use similar pictorial representations as in Year 4 and 5	Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits   When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20. This is known as a placeholder.

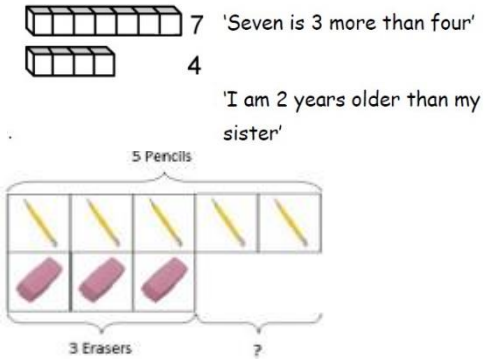
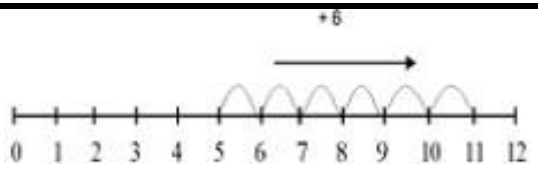
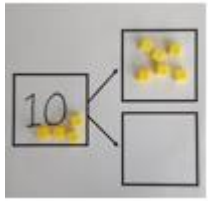
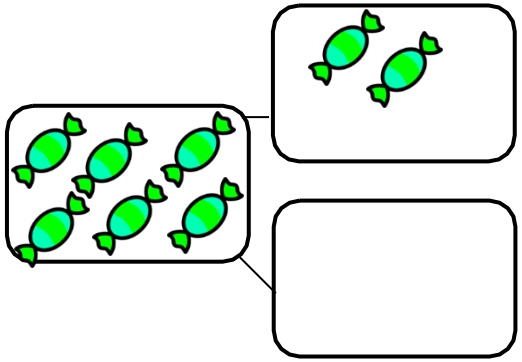
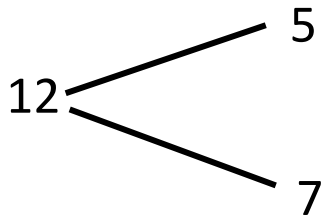
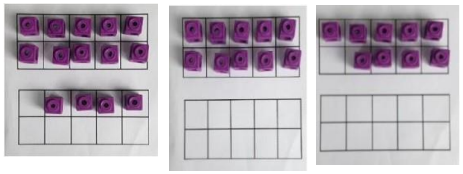
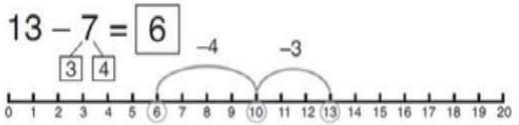
Reception - Subtraction

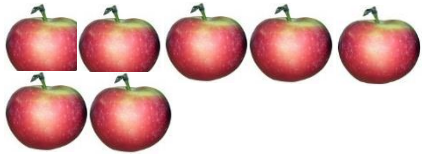
Objective/strategy	Concrete	Pictorial	Abstract
Early Learning Goal: Using quantities and objects, children subtract two single-digit numbers and count on or back to find the answer.			
Taking away	9-4 =  Taking away after counting out practical equipment. . Children would be encouraged to physically remove these using touch counting.  By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer. **Those who are ready** may record their own calculations	 8 − 4 = __ 0 1 2 3 4 5 6 7 8 9 10	8-5 = 3

			5-3=2

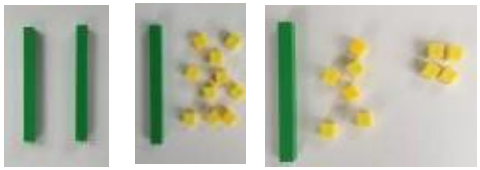
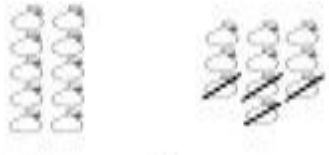
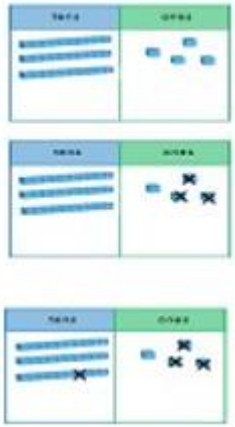
Year 1 - Subtraction

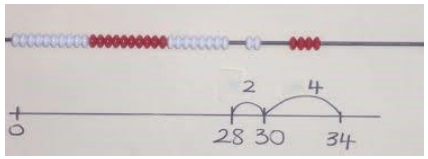
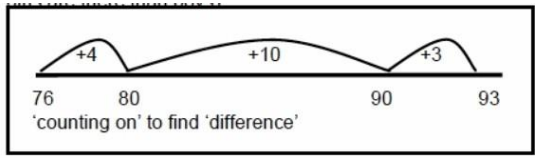
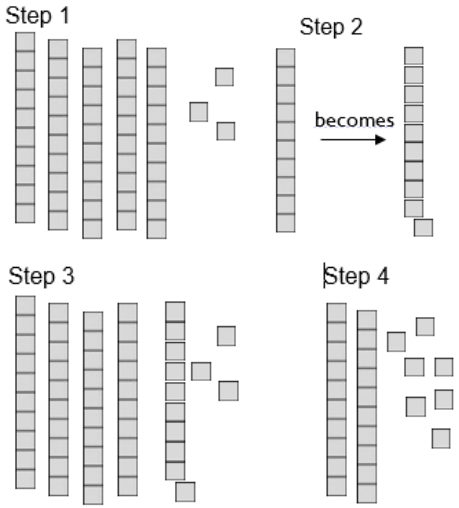
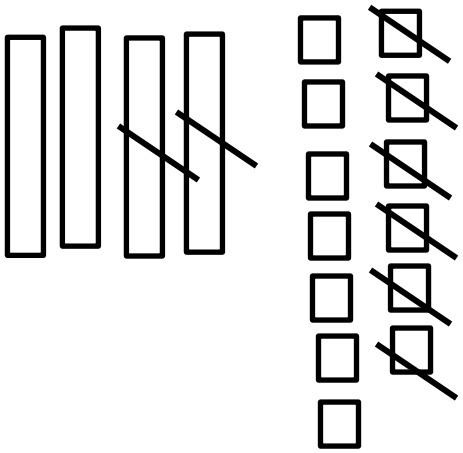
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).			
Taking away ones.	Use and move physical objects, counters, cubes etc to show how objects can be taken away. 6—4 = 2  4 -2 =2 	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	**7—4 = 3** **16—9 = 7**
Counting back	Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards. 	Count back in ones using a number line. 	Put 13 in your head, count back 4. What number are you at?
Find the Difference	Compare objects and amounts	Count on using a number line to find the difference.	Hannah has12 sweets and her sister has 5. How many more does Hannah have than her sister?

	 Lay objects to represent bar model.		
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	Link to addition. Use part part whole model to model the inverse.  If 10 is the whole and 6 is one of the parts, what is the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5. 14—9 	Jump back 3 first, then another 4. Use ten as the stopping point. 13—7 	How many do we take off first to get to 10? How many left to take off? 16—8

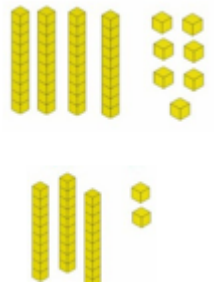
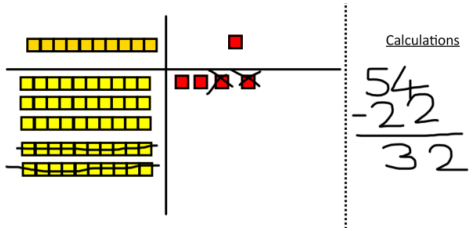
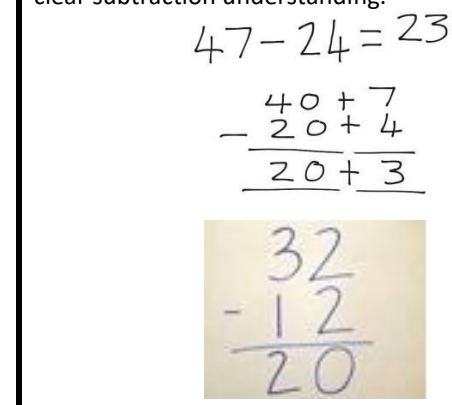
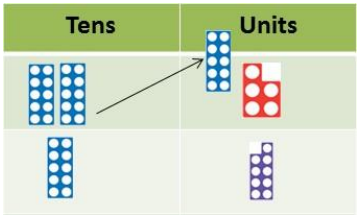
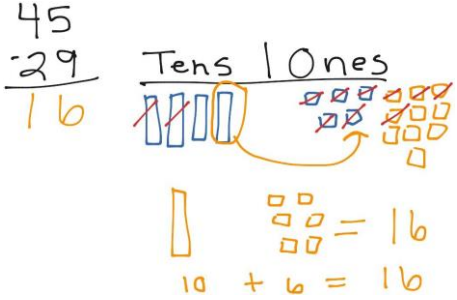
Bar model Including the inverse operations.	 5—2 = 3		8 | 2 10 = 8 + 2 10 = 2 + 8 10 – 8 = 2 10 – 2 = 8

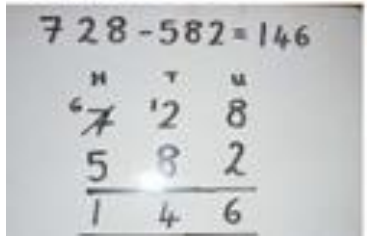
Year 2 - Subtraction

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers. Subtract numbers with up to three digits, using formal written method of columnar subtraction.			
Regroup a ten into ten ones	Use a PV chart to show how to change a ten into ten ones, use the term 'take and make' 	 20 — 4 =	20—4 = 16
Partitioning to subtract without regrouping. 'Friendly numbers'	Use Dienes to show how to partition the number when subtracting without regrouping. 34—13 = 21 	Children draw representations of Dienes and cross off. 43—21 = 22 	43—21 = 22

Make ten strategy Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.	Use a bead bar or bead strings to model counting to next ten and the rest. 34—28 	Use a number line to count on to next ten and then the rest. 	Build on to column subtraction with 2 digits and reinforce regrouping 8 ~~9~~ 14 -3 6 5 8
Regrouping	**53-26** 		4 ~~5~~ 1 3 2 6 2 7

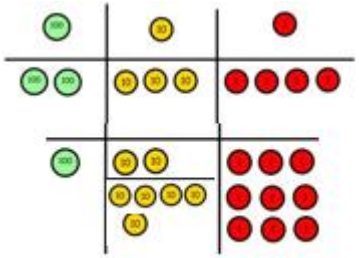
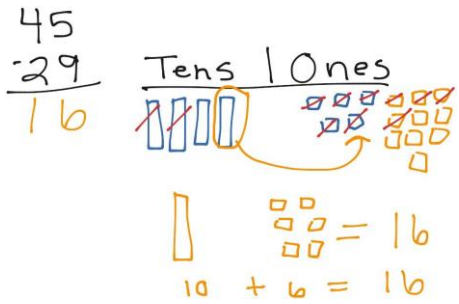
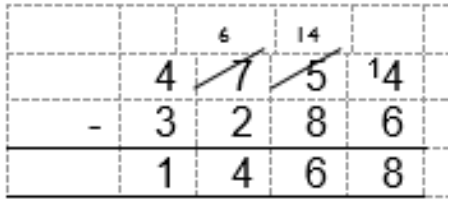
Year 3 - Subtraction

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Subtract numbers with up to three digits, using formal written method of columnar subtraction using the same methods learnt in Year 2.			
Column subtraction without regrouping (friendly numbers)	Use base 10 or Numicon to model 47—32 	Draw representations to support better understanding using images 	Intermediate step may be needed to lead to clear subtraction understanding. 
Column subtraction with regrouping	Begin with base 10 or Numicon. Move to place value counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange 	Children may draw base 10 or place value counters and cross off to work out the subtraction 	Begin by partitioning into place value columns if necessary for children's better understanding  By the end of Year 3 children should be be able to use formal method for 3 digit numbers

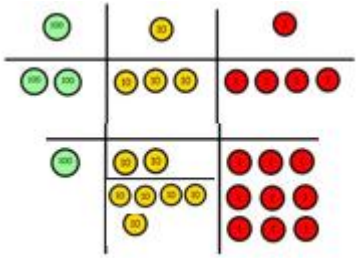
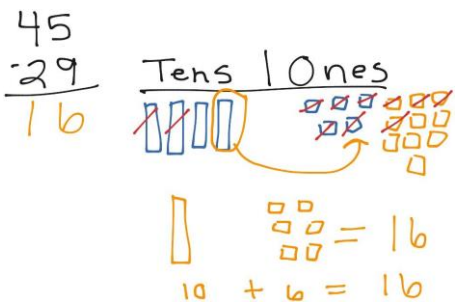
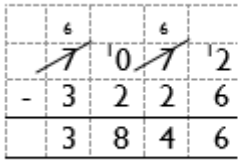
			

728 - 582 = 146
H T U
6 12 8
5 8 2
1 4 6

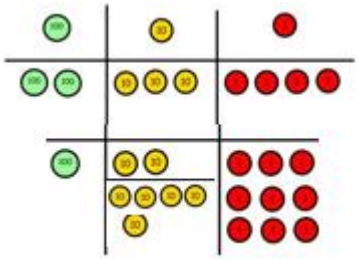
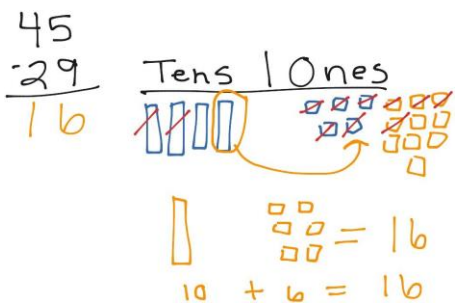
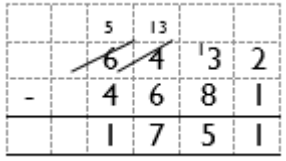
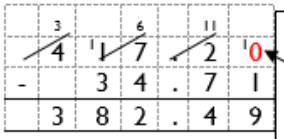
Year 4 - Subtraction

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Subtract numbers with up to 4 digits and decimals with one decimal place using the formal written method of columnar subtraction.			
Subtracting tens and ones subtract with up to 4 digits. Introduce decimal subtraction through context of money	Model process of exchange using Numicon, base ten and then move to place value counters. 234 – 179 	Draw and show excahnge just as in Year 3 	By the end of Y4, children should be using the written method confidently and with understanding. They will also be subtracting: -numbers with different numbers of digits, understanding the place value; -decimals with one decimal place, knowing that the decimal points line up under one another. 

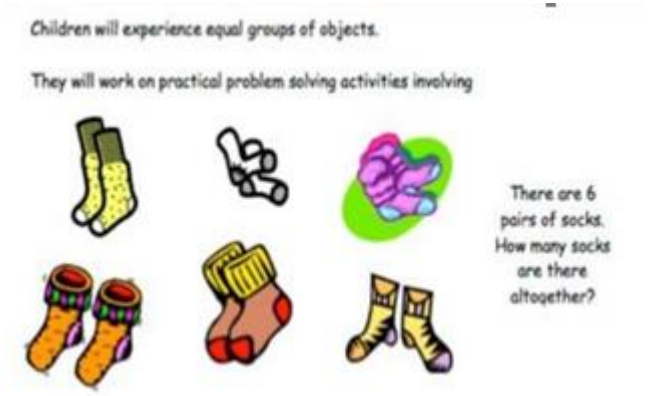
Year 5 - Subtraction

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Subtract whole numbers with more than 4 digits and decimals with two decimal places, using the formal written methods (columnar subtraction).			
Subtract with at least 4 digits, including money and measures. **They will also be subtracting:** -Numbers with different numbers of digits, understanding the place value; -Decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another. -Amounts of money and measures, including those where they have to initially convert from one unit to another	Same as Year 4 Model process of exchange using Numicon, base ten and then move to PV counters. 234 – 179 	Draw and show excahnge just as in Year 3 	 

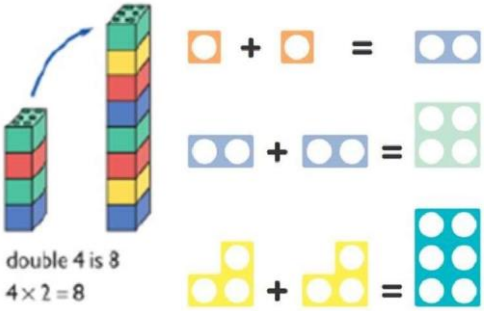
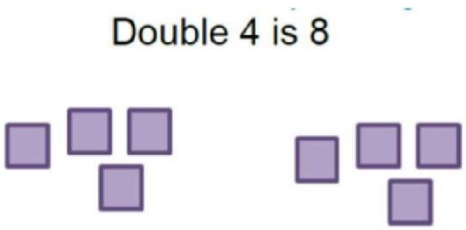
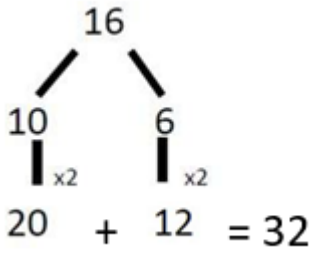
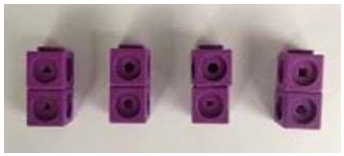
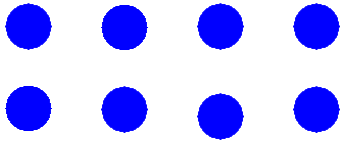
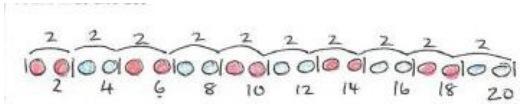
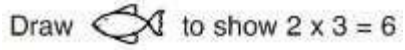
Year 6 - Subtraction

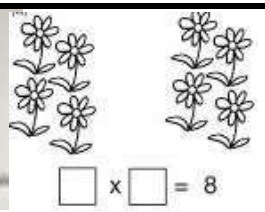
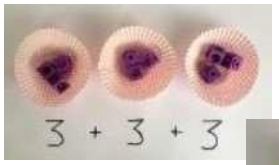
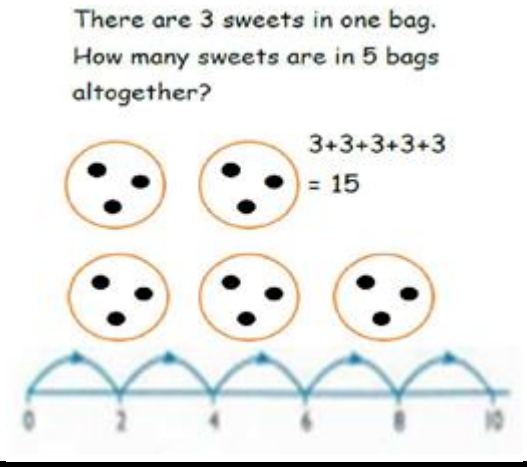
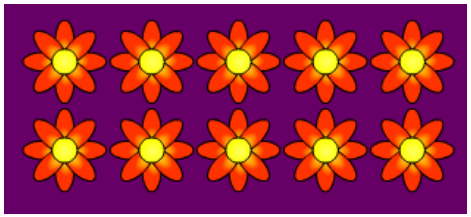
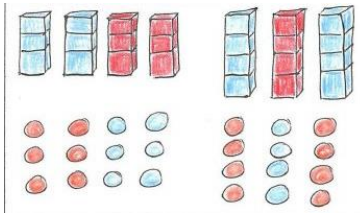
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Subtract whole numbers and decimals using formal written methods (columnar subtraction).			
Subtract with increasingly large and more complex numbers and decimal values.	Just as Year 4 Model process of exchange using Numicon, base ten and then move to place value counters. 234 – 179 	Draw and show excahnge just as in Year 3 	  When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

Reception - Multiplication

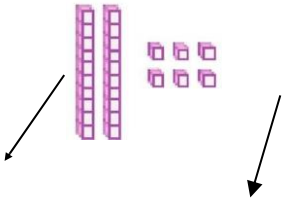
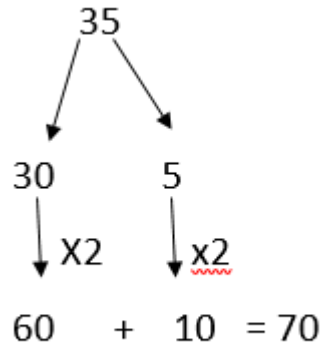
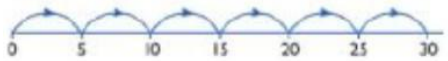
Objective/strategy	Concrete	Pictorial	Abstract
Early learning goal: Children solve problems, including doubling.			
Experiencing equal groups of objects They will think about doubling when solving practical problems.	Children use variety of concrete objects including cubes, animals, blocks, beads etc	 	2 + 2 = 4

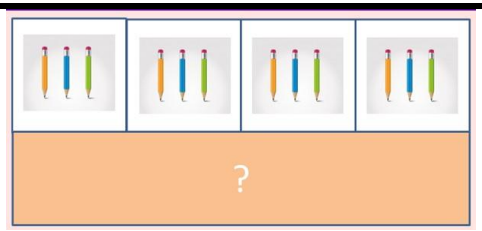
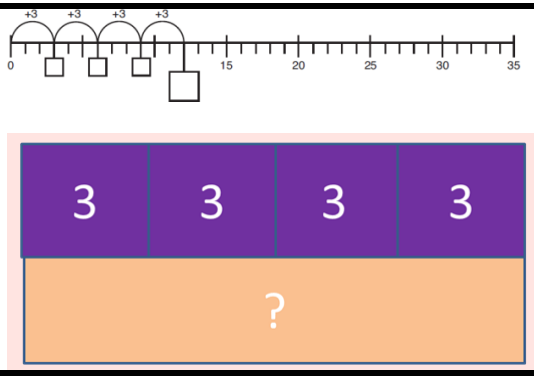
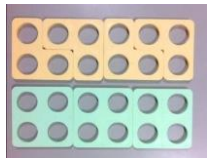
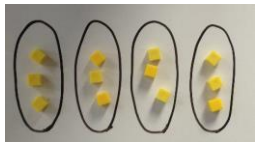
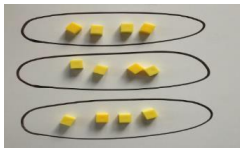
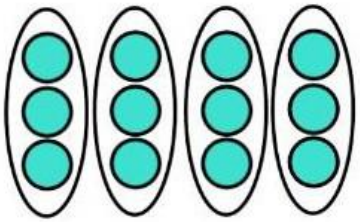
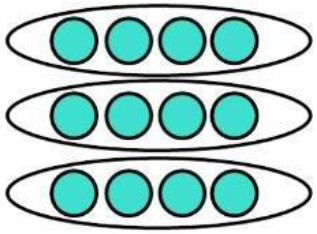
Year 1 – Multiplication

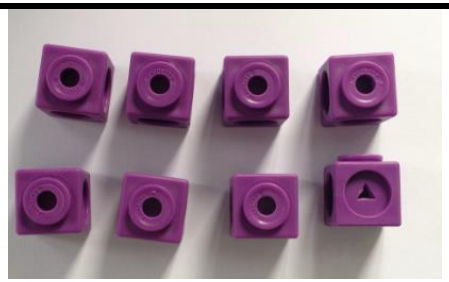
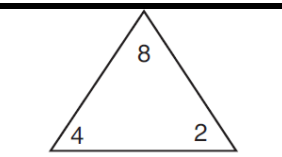
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.			
Doubling	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling 	Draw pictures to show how to double numbers 	Partition a number and then double each part before recombining it back together. 
Counting in multiples (2s, 5s, 10s)	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	Children make representations to show counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total	Use manipulatives to create equal groups.	Draw and make representations 	2 x 4 = 8

	 		
Repeated addition	Use different objects to add equal groups   	Use pictorial representations and number line to solve problems 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show under-standing 	3 x 2 = 6 2 x 5 = 10

Year 2 - Multiplication

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Calculate mathematical statements for multiplication (using repeated addition) and write them using the multiplication (x) and equals (=) signs.			
Doubling	Model doubling using dienes and place value counters. Double 26  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40 	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30

			$4 \times 3 = \square$
Multiplication is commutative	Create arrays using counters and cubes and Numicon.   Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse			Show all 8 related fact family sentences.

		 □ × □ = □ □ × □ = □ □ ÷ □ = □ □ ÷ □ = □	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2

Year 3 - Multiplication

Objective/strategy	Concrete	Pictorial	Abstract

End of Year Objective:
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.

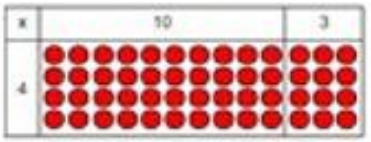
Multiply 2 digit numbers by 1 digit numbers

Grid method

It is important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

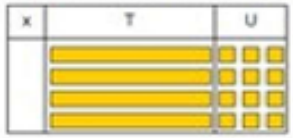
From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the

Show the links with arrays to first introduce the grid method.



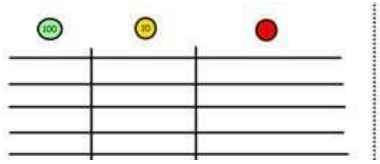
4 rows of 10
4 rows of 3

Move onto base ten to move towards a more compact method.

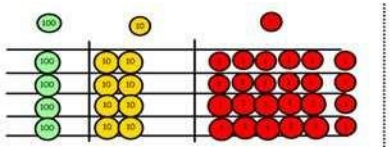


4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows

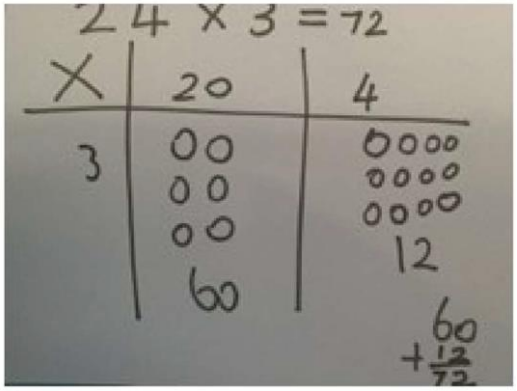


Calculations
4 x 126

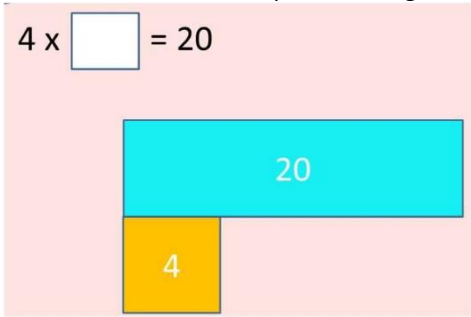


Calculations
4 x 126

Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.



Bar model are used to explore missing numbers



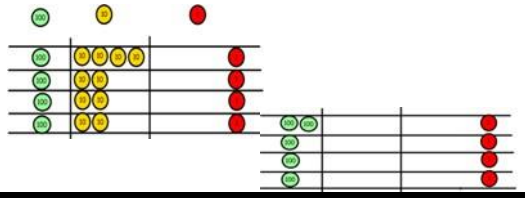
Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

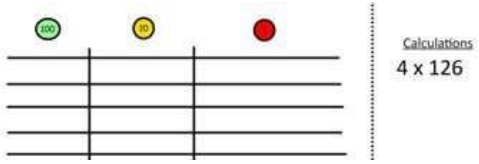
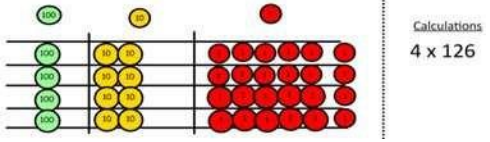
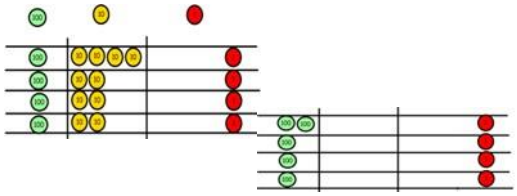
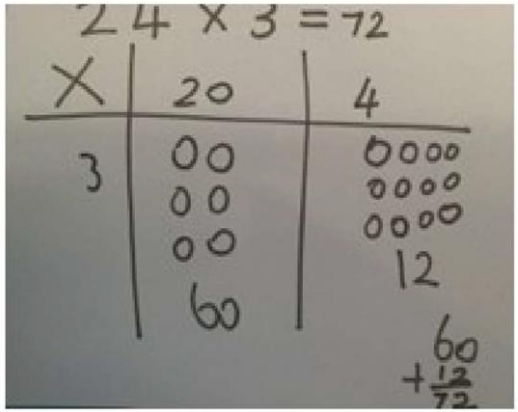
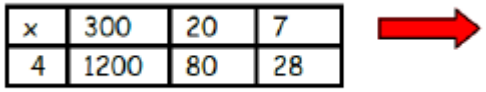
210 + 35 = 245

Move forward to the formal written method:

$$\begin{array}{r} 35 \\ \times\ 7 \\ \hline 245 \\ \hline 3 \end{array}$$

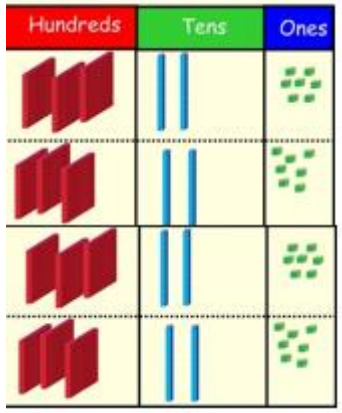
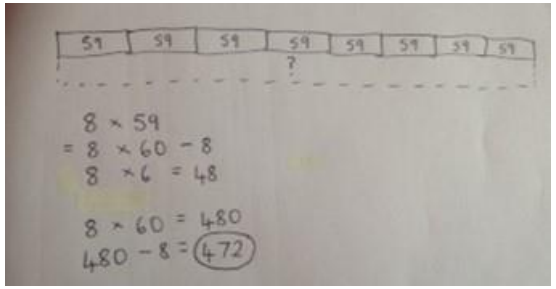
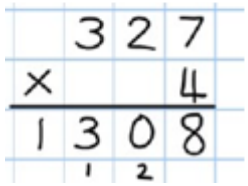
addition mentally, they may do so.	Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer. 		

Year 4 - Multiplication

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Multiply two-digit and three-digit numbers by a one-digit number using formal written method (expanded vertical column)			
Grid method recap from year 3 for 2 digits x 1 digit Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)	Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows  Calculations 4 x 126  Calculations 4 x 126 Fill each row with 126. Add up each column, starting with the ones making any exchanges needed Then you have your answer. 	Children can represent their work with place value counters in a way that they understand. They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below. 	Start with multiplying by one digit numbers and showing the clear addition alongside the grid. **210 + 35 = 245**
Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642 It is important at this stage that they always multiply the ones first. The corresponding long multiplication is	The grid method my be used to show how this relates to a formal written method. 	This may lead to a compact method.

×	30	5
7	210	35

x	300	20	7
4	1200	80	28

	modelled alongside 	Bar modelling and number lines can support children when solving problems with multiplication alongside the formal written methods. 	327 x 4 28 80 1200 1308  Children should only be expected to move towards compact method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value.

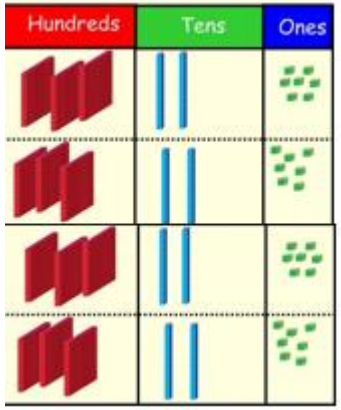
Year 5 - Multiplication

Objective/strategy	Concrete	Pictorial	Abstract

End of Year Objective:
Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.

Column multiplication

Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642
It is important at this stage that they always multiply the ones first.
The corresponding long multiplication is modelled alongside



The grid method my be used to show how this relates to a formal written method.

x	300	20	7
4	1200	80	28

Bar modelling and number lines can support children when solving problems with multiplication alongside the formal written methods.

7 x 43

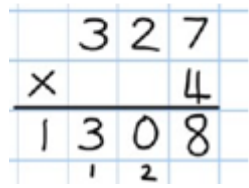
43	43	43	43	43	43	43

7 x 40 = 280
7 x 3 = 21

280 + 21 = 301

This may lead to a compact method.

$$\begin{array}{r} 327 \\ \times \quad 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$$

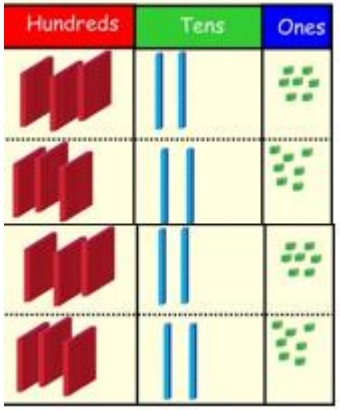
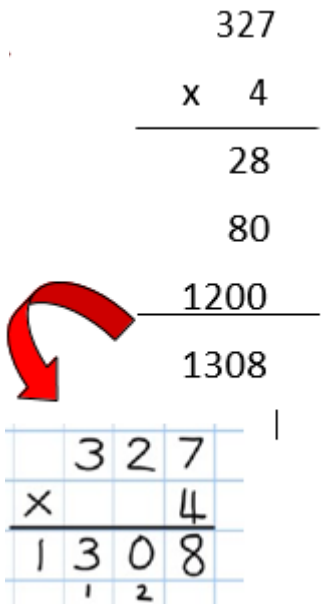


Children should only be expected to move towards compact method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value.

Column multiplication	Manipulatives may still be used with the corresponding long multiplication modelled alongside.	368 x 6 x 300 60 8 1800 + 360 6 1 800 360 48 + 48 2208	Th H T U 3 6 8 x 6 4 8 (8 x 6) 3 6 0 (60 x 6) + 1 8 0 0 (300 x 6) 2 2 0 8 Th H T U 3 6 8 x 6 2 2 0 8
		693 x 24 x 600 90 3 20 12000 1800 60 = 13 860 4 2400 360 12 = 2 772 + 16 632	Children should only be expected to move towards this next method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value. 693x24 Step 1 TTh Th H T U 6 9 3 x 2 4 2 7 7 2 (693x 4)

			Step 2 TTh Th H T U 6 9 3 x 2 4 2 7 7 2 (693 x 4) + 1 3 8 6 0 (693 x 20)
		Continue to use bar modelling to support problem solving 7 x 43 43 | 43 | 43 | 43 | 43 | 43 | 43 **7 x 40 = 280** **7 x 3 = 21** **280 + 21 = 301**	Step 3 TTh Th H T U 6 9 3 x 2 4 2 7 7 2 (693 x 4) + 1 3 8 6 0 (693 x 20) 1 6 6 3 2

Year 6 - Multiplication

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication. Multiply one-digit numbers with up to two decimal places by whole numbers.			
Column multiplication As in year 5	Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642 It is important at this stage that they always multiply the ones first. The corresponding long multiplication is modelled alongside 	The grid method my be used to show how this relates to a formal written method. x 300 20 7 4 1200 80 28 Bar modelling and number lines can support children when solving problems with multiplication alongside the formal written methods. 7 x 43 43 43 43 43 43 43 43 **7 x 40 = 280** **7 x 3 = 21** **280 + 21 = 301**	This may lead to a compact method.  Children should only be expected to move towards compact method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value.

<table>
<tr>
<td>Column multiplication</td>
<td>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</td>
<td>368 x 6
<table><tr><td>x</td><td>300</td><td>60</td><td>8</td><td></td><td>1800</td></tr><tr><td>6</td><td>1 800</td><td>360</td><td>48</td><td>+
+</td><td>360
48</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td>2208</td></tr></table></td>
<td>Th H T U
3 6 8
x 6
4 8 (8 x 6)
3 6 0 (60 x 6)
+ 1 8 0 0 (300 x 6)
2 2 0 8

Th H T U
3 6 8
x 6
2 2 0 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>693 x 24
<table><tr><td>x</td><td>600</td><td>90</td><td>3</td><td></td></tr><tr><td>20</td><td>12000</td><td>1800</td><td>60</td><td>= 13 860</td></tr><tr><td>4</td><td>2400</td><td>360</td><td>12</td><td>= 2 772 +</td></tr><tr><td></td><td></td><td></td><td></td><td>16 632</td></tr></table></td>
<td>Children should only be expected to move towards this next method if they have a secure understanding of place value. It is difficult to explain the compact method without a deep understanding of place value.

693x24

Step 1
TTh Th H T U
6 9 3
x 2 4
2 7 7 2 (693x 4)

Step 2
TTh Th H T U
6 9 3
x 2 4
2 7 7 2 (693 x 4)
+ 1 3 8 6 0 (693 x 20)</td>
</tr>
</table>

		Continue to use bar modelling to support problem solving 7 x 43 **43** **43** **43** **43** **43** **43** **43** **7 x 40 = 280** **7 x 3 = 21** **280 + 21 = 301**	Step 3 TTh Th H T U 6 9 3 x 2 4 2 7 7 2 (693 x 4) + 1 3 8 6 0 (693 x 20) 1 6 6 3 2

By the end of year 6, children should be able to use the grid method to multiply any number by a two-digit number. They could also develop the method to be able to multiply decimal numbers with up to two decimal places, but having been introduced to expanded and compact vertical methods in Year 5, it may be appropriate to use the expanded vertical method when introducing multiplication involving decimals.

4.92 x 3

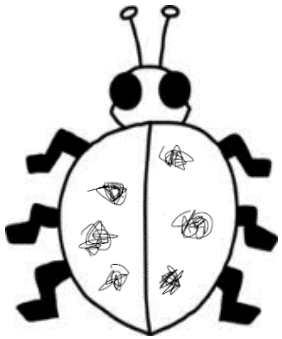
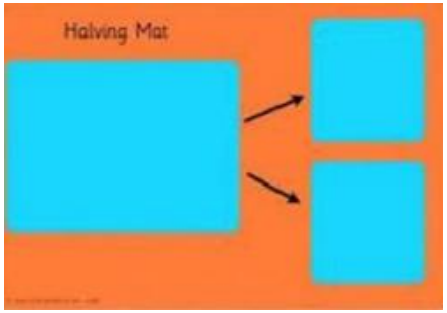
T U . t h
4 . 9 2
x 3
0 . 0 6 (0.02 x 3)
2 . 7 (0.9 x 3)
+ 1 2 (4 x 3)
1 4 . 7 6



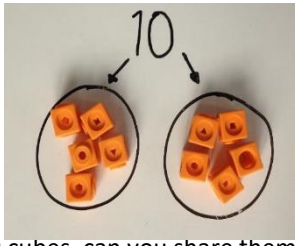
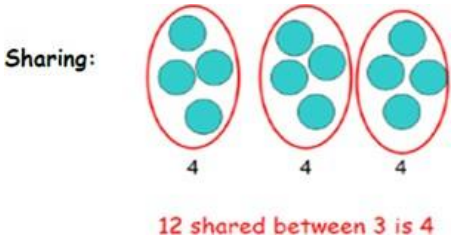
T U . t h
4 . 9 2
x 3
1 2 4 . 7 6

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

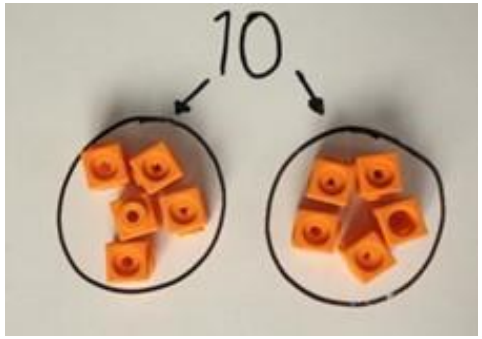
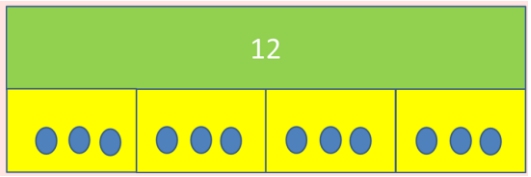
Reception - Division

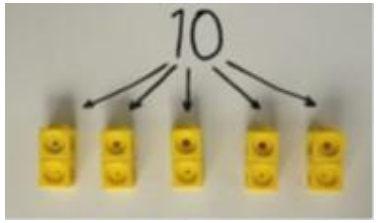
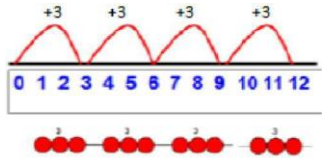
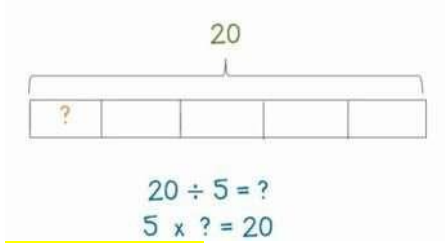
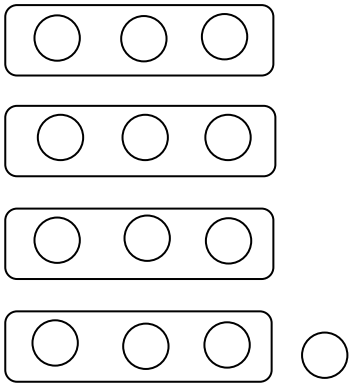
Objective/strategy	Concrete	Pictorial	Abstract
Early Learning Goal: Children solve problems, including halving and sharing.			
Sharing practical objects. Hearing and being exposed to language to describe half and seeing visual representations	Children should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins, which are arrays.  	A child's jotting showing halving six spots between two sides of a ladybird. 	 

Year 1 - Division

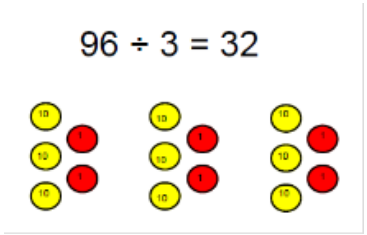
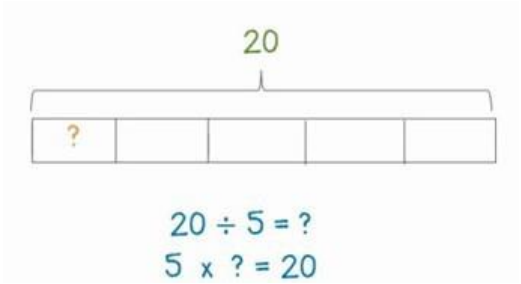
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.			
Children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out. Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.			
Division as sharing	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4 	12 shared between 3 is 4

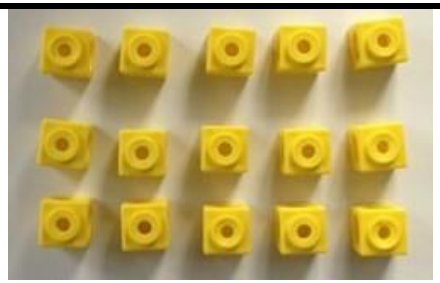
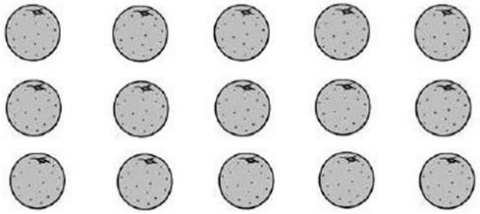
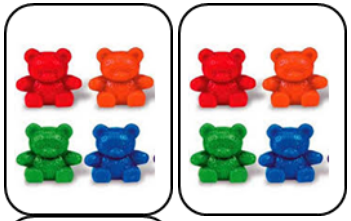
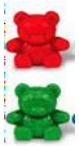
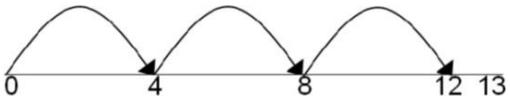
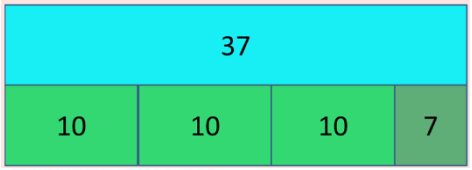
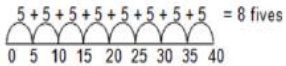
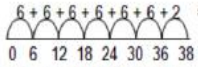
Year 2 - Division

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.			
Division as sharing	I have 10 cubes, can you share them equally in 2 groups?  	Children use pictures or shapes to share quantities  8 ÷ 2 = 4 Children use bar modelling to show and support understanding. 12 ÷ 4 = 3  	12 ÷ 3 = 4

Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  **12 ÷ 3 = 4** Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  **Extension for GDS** Children should also continue to develop their knowledge of division with remainders 13 ÷ 4 = 	15 ÷ 5 = 3 Divide 15 into 5 groups. How many are in each group? 13 ÷ 4 = 3 remainder 1

Year 3 - Division

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods**.**			
Division as grouping	Use cubes, counters, objects or place value counters to aid understanding.  24 divided into groups of 6 = 4 	Continue to use bar modelling to aid solving division problems. 	How many groups of 6 in 24? 24 ÷ 6 = 4
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences	Find the inverse of multiplication and division sentences by creating eight linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7

			4 = 28 ÷ 7 7 = 28 ÷ 4
Division with remainders	14 ÷ 3 = Divide objects between groups and see how much is left over   	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders.  Example without remainder: 40 ÷ 5 Ask "How many 5s in 40?"  Example with remainder: 38 ÷ 6  For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.	Complete written divisions and show the remainder using r. 

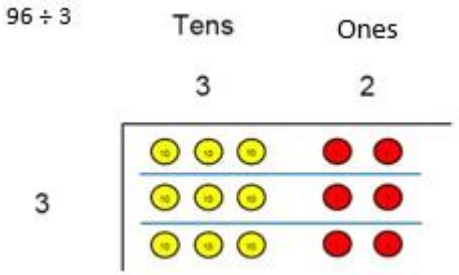
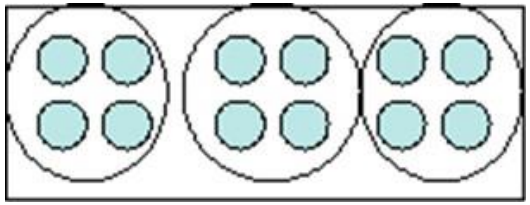
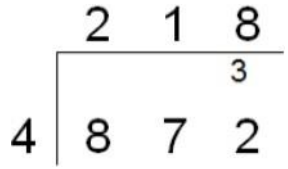
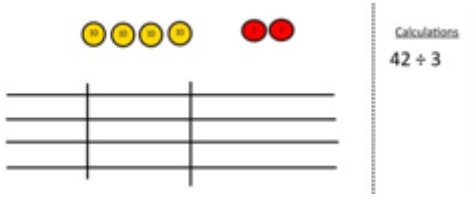
The children may begin to divide numbers where the dividend is beyond their tables knowledge.
E.g. 56 ÷ 4 = 14
Some more able children may be encouraged to make use of known mental facts for this. In this case thinking of 56 as being made up of 40 and 16 and knowing that 40 ÷ 4 = 10 and 16 ÷ 4 = 4

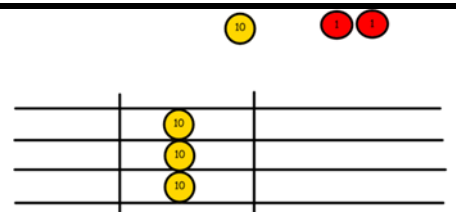
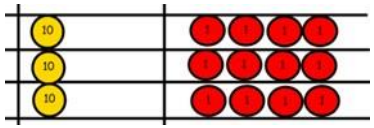
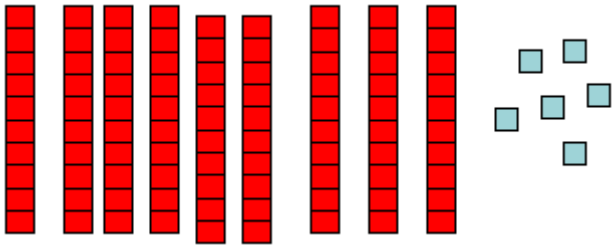
The teacher will use their professional judgement to make a decision about when children may see the traditional layout of division:

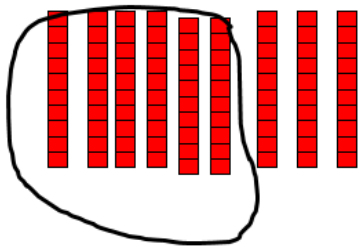
$$\begin{array}{r} 14 \\ 4\overline{)56} \end{array}$$

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Year 4 - Division

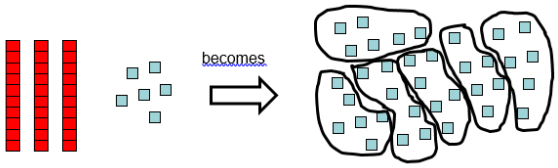
Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.			
Divide at least 3 digit numbers by 1 digit. Short Division	96 ÷ 3  Use place value counters to divide using the bus stop method alongside	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.  Encourage them to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainder. 
	42 ÷ 3=  Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.		

	 We exchange this ten for ten ones and then share the ones equally among the groups.  We look how much in 1 group so the answer is 14.		
	96 ÷6 = Use dienes  How many groups of 6 can be made out of the tens (without changing their form)?	Children can draw lines and circles to represent tens and ones	$\begin{array}{r} 16 \\ 6\overline{)96} \end{array}$ This will progress onto dividing a 3 digit number by a 1 digit number.



$$\begin{array}{r} 1 \\ 6\overline{)96} \end{array}$$

There are 3 tens left over so they need to be exchanged for units (ones) leaving:

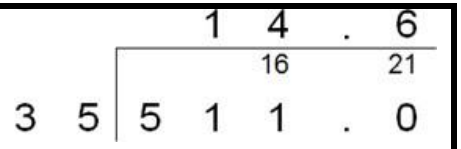


How many groups of 6 can be made out of the 36 units (ones)?

$$\begin{array}{r} 16 \\ 6\overline{)96} \end{array}$$

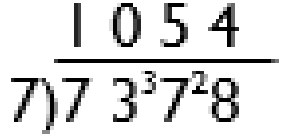
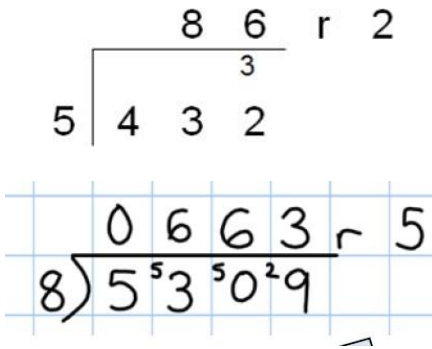
Year 5 - Division

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.			
Divide 4 digit by 1 digit number Short division	Dienes may continue to be used to model the method of division where necessary. Children will develop their skills with increasingly big numbers including calculations where there is exchange from the first number and then where there is exchange within the number. I.e. no carry from thousands to hundreds in the example underneath. $4\overline{)5^{1}0^{2}84}$ = 1271 $7\overline{)7\,3^{3}7^{2}8}$ = 1054	Encourage them to count in multiples to divide more efficiently. Children may draw representations where necessary	$7\overline{)7\,3^{3}7^{2}8}$ = 1054 Move onto divisions with a remainder. $5\overline{)4\,3\,2}$ = 86 r 2 (3) $8\overline{)5^{5}3^{5}0^{2}9}$ = 0663 r 5 Finally move into decimal places to divide the total accurately.

Children should be secure in using the short division by the end of Year 5.
Introduce long division for groups of children who might benefit from it for better understanding.

Year 6 - Division

Objective/strategy	Concrete	Pictorial	Abstract
End of Year Objective: Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context. Use written division methods in cases where the answer has up to two decimal places.			
Short division	Dienes may continue to be used to model the method of division where necessary.	Encourage them to count in multiples to divide more efficiently. Children may draw representations where necessary	 Move onto divisions with a remainder.  Finally move into decimal places to divide the total accurately.

To develop the formal method further, it should be extended to include dividing a four-digit number by a two-digit number			14.6 16 21 35 ⟌ 511.0 6367 ÷ 28 227r11 28 ⟌ 6367 56 76 56 207

LONG DIVISION

Step 1—a remainder in the ones

```
    h  t  o
    0  4  1 R1
  ---------
4 ) 1  6  5
```

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

```
     th  h  t  o
     0  4  0  0 R7
  ------------
8 ) 3  2  0  7
```

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times (3,200 ÷ 8 = 400)
8 goes into 0 zero times (tens).
8 goes into 7 zero times, and leaves a remainder of 7.

```
   h  t  o
      0  6  1
   ---------
4 ) 2  4  7
   - 4
   -----
      3
```

When dividing the ones, 4 goes into 7 one time. Multiply 1 × 4 = 4, write that four under the 7, and subract. This finds us the remainder of 3.

Check: 4 × 61 + 3 = 247

```
   th  h  t  o
       0  4  0  2
   ------------
4 ) 1  6  0  9
      - 8
      -----
         1
```

When dividing the ones, 4 goes into 9 two times. Multiply 2 × 4 = 8, write that eight under the 9, and subract. This finds us the remainder of 1.

Check: 4 × 402 + 1 = 1,609

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 2)5 8	t o 2 2)5 8 - 4 1	t o 2 9 2)5 8 - 4 ↓ 1 8
Two goes into 5 two times, or 5 tens ÷ 2 = 2 whole tens -- but there is a remainder!	To find it, multiply 2 × 2 = 4, write that 4 under the five, and subtract to find the remainder of 1 ten.	Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
t o 2 9 2)5 8 - 4 1 8	t o 2 9 2)5 8 - 4 1 8 - 1 8 0	t o 2 9 2)5 8 - 4 1 8 - 1 8 0
Divide 2 into 18. Place 9 into the quotient.	Multiply 9 × 2 = 18, write that 18 under the 18, and subtract.	The division is over since there are no more digits in the dividend. The quotient is 29.

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
h t o 1 2)2 7 8 Two goes into 2 one time, or 2 hundreds ÷ 2 = 1 hundred.	h t o 1 2)2 7 8 -2 0 Multiply 1 × 2 = 2, write that 2 under the two, and subtract to find the remainder of zero.	h t o 1 8 2)2 7 8 -2 ↓ 0 7 Next, drop down the 7 of the tens next to the zero.
Divide.	**Multiply & subtract.**	**Drop down the next digit.**
h t o 1 3 2)2 7 8 -2 0 7 Divide 2 into 7. Place 3 into the quotient.	h t o 1 3 2)2 7 8 -2 0 7 -6 1 Multiply 3 × 2 = 6, write that 6 under the 7, and subtract to find the remainder of 1 ten.	h t o 1 3 2)2 7 8 -2 0 7 -6 1 8 Next, drop down the 8 of the ones next to the 1 leftover ten.
1. Divide.	**2. Multiply & subtract.**	**3. Drop down the next digit.**
h t o 1 3 9 2)2 7 8 -2 0 7 -6 1 8 Divide 2 into 18. Place 9 into the quotient.	h t o 1 3 9 2)2 7 8 -2 0 7 -6 1 8 -1 8 0 Multiply 9 × 2 = 18, write that 18 under the 18, and subtract to find the remainder of zero.	h t o 1 3 9 2)2 7 8 -2 0 7 -6 1 8 -1 8 0 There are no more digits to drop down. The quotient is 139.